Math 2263, Quiz 10
You must show all work for full credit, you have 15 min to finish it.

1.(4 pt) Find the Jacobian of the transformation: $x = 4u + v, y = 2u - v$.
Solution: The Jacobian equals to $\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u} = 4 \times (-1) - 1 \times 2 = -6$.

2.(5 pt) Find the image of the set S under the given transformation:
S is the square bounded by the line $u = 0, u = 1, v = 0, v = 1$; $x = v, y = uv$.

Solution: The transformation maps the boundary to the boundary. $u = 0$ will be mapped to $y = 0$, $v = 0$ will be mapped to the point $(0, 0)$, $v = 1$ will be mapped to $x = 1$. For $u = 1$, the image will be $x = v, y = v$, which is just the line $x = y$. So the image of S under the transformation is just the triangular region bounded by $x = 1, y = 0$ and $x = y$.

3.(6 pt) Use the given transformation to evaluate the integral $\iint_R x^2 dA$ where R is the region bounded by the ellipse $9x^2 + 4y^2 = 36; x = 2u, y = 3v$.

Solution: The Jacobian of the transformation is $2 \times 3 = 6$. Under this transformation, the region R will be mapped to $S = \{(u, v) \mid 36u^2 + 36v^2 = 36\} = \{(u, v) \mid u^2 + v^2 = 1\}$, which is just the region enclosed by unit circle. Our integral is equal to $\iint_S 6(2u)^2 dA = \iint_S 24u^2 dA$.
Use the polar coordinates, $u = r\cos(\theta), v = r\sin(\theta)$, then $r \in [0, 1], \theta \in [0, 2\pi]$.
The integral will be $\int_0^{2\pi}\int_0^1 24r^3\cos^2(\theta)drd\theta = \int_0^{2\pi} 6\cos^2(\theta)d\theta = \int_0^{2\pi} 6(\frac{\cos(2\theta)+1}{2})d\theta = \int_0^{2\pi} 3\cos(2\theta) + 3d\theta = 6\pi$.